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Assessing & Developing
Math Concepts

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Kathy Richardson is the author and developer of the Assessing Math Concepts (AMC) series of assessments and the Developing Number Concepts (DNC) series for Kindergarten through Second Grade Mathematics. Kathy, Program Director for Math Perspectives, is one of the most respected early childhood mathematics educators. Kathy answers questions from teachers across the country who are using AMC and DNC.

If you have questions for Kathy, please send them to Math Perspectives at info@mathperspectives.com.

ASSESSING MATH CONCEPTS:
Two Digit Addition and Subtraction

Q Hello. A question was brought to me regarding the 2 digit addition and subtraction assessment. The concern was as we have shifted away from the traditional algorithm and students are more readily using mental math strategies like partial sums or adding up in parts, students would be penalized for using number sense based strategies rather than regrouping.

They feel that the phrasing used when identifying the strategy students use "make/adds a ten with ease" or "makes/adds ten with effort" are only referring to making a ten by regrouping. Is this true?

If a student were to say " I added the tens to get 30 and then I broke the 6 in to 2, 2, and 2 and added one of the 2s to the 8 to get another 10 - which makes 40 and then there are still another 4. So the total is 44."

How would you score this answer?

Thank you for your support. Warm regards. -- *Stafford, VA*

PS - I am also not 100% sure why the first question is how many tens and how many ones. Wouldn't we get the same information if we asked for the total first and then asked for the how and listened for the strategy? My own son (who has incredible number sense), stumbled on that saying that 50 tens and 3 ones when he was trying to describe 53. If students are using a partial sum approach, their thinking will be in expanded form rather than 10s and 1s.

A You are asking really good questions. I will do my best to answer.

Your example of a way that a student might solve the problem 28 + 16 is one I would describe as "Makes and adds tens" and I am assuming this was "with ease".
I think of combining 2 digit numbers as essentially making all the tens that can be made and then adding on the ones that are left. So "adding" the tens to get to 30 and "making" another ten by adding 2 to the 8 is "Making and adding tens".

Another way that a child might do the problem is by breaking up 16 into 1 ten and 6. Then adding 28 + 10 to get 38. (Adding tens) and taking 2 from the 6 to make another ten (makes tens) with 4 left over.

What I don't want to encourage is the traditional algorithm. I am asking the question in such a way that the algorithm doesn't answer the question. It is not the way children add if they are thinking about the tens and ones in the model. My experience is when I show a model of tens and ones and ask children to add to the model (mentally), they automatically make all the tens they can and add on the ones. They will generally do this even if they have not had much experience thinking with the numbers. Many then revert to the algorithm when they see the symbolic problem. Using the algorithm is not considered Ready to Apply when they are asked to think about the model.

When I present the symbolic problem, I felt it might be reasonable for a student to solve that using the algorithm if that is what the teacher requires when giving symbolic problems. But I only give them credit for that if they can demonstrate what is happening with the numbers when they do the algorithm. What I hope they will do is the same kind of thinking they do when the model is present.

The reason I ask "how many tens and ones?" is because I think the kids will be more numerically powerful if they "think in tens." I want them to think of numbers as the "units" they are composed of - so 53 is 5 tens and 3. I want them to think of 1 ten rather than 10 ones. Then later I want them to think of 1 thousand rather than 1000 ones and so on. It is easier to conceptualize adding 3 tens to a number than adding 30 - or later adding 6 hundreds to a number than adding 600.

I want them to easily distinguish between the value of a number and the structure of the number. So, a child who solves the problem by thinking of the number of tens will naturally describe how many tens there will be and how many ones will be left so I like to encourage that and have them report it that way. If the student you described was "thinking in tens", he might say, " I added the 2 tens and 1 ten and that was 3 tens. Then I broke off 2 of the 6 to make another ten and now I have 4 tens and 4 leftover. That makes 44.

It would not be unusual for a child like your son to stumble at first if it is language he is not used to using... but I think it would be useful for him to have some practice and be flexible enough to answer with either the number of tens or the value of the tens depending on the situation. In any case, there is no penalty if a child says 50 tens. The teacher is just supposed to say it back to him, "50 tens?" If he understands tens and ones, he would self-correct.

I believe "thinking in tens" helps children develop the kind of thinking described here more easily. For example, if I asked a child, "How many would there be if we had 26 and we added 7 more?", the child might break up the 7 to get 4 to finish the ten and then have 3 left over for a total of 3 tens and 3 ones. Then if I ask, "How many would there be if we added 17 to the 26?", children who "think in tens" are able to say almost instantly, "43, because it is just the same but now we have one more ten.

I hope this helps. If you have more questions about this, please let me know. ~ *Kathy*

CRITICAL LEARNING PHASES and RANGES

***Background information**: The following question comes from a math coordinator who has the new book Number Talks in the Primary Classroom. The mathematics children will work on during Number Talks has been organized into Instructional Ranges. The Critical Learning Phases (described in How Children Learn Number Concepts: A Guide to the Critical Learning Phases) are listed with those Ranges related to them.*

Q Do they need to achieve the Critical Learning Phase to be in that Instructional Range or are they developing the phase during that range? - *Wyoming, USA*

A The short answer is that children will be developing the Critical Learning Phase(s) while working in the related Range(s). But there is more to think about when considering how they are related.

1. Each Critical Learning Phase describes insights that children need to have in order to benefit from instruction.

Let's consider the following Critical Learning Phase as an example:
"Recognizes and describes parts contained in larger numbers."

Before children can describe parts, they have to be able to "see parts". When children still think of numbers as a collection of units, they can't see parts. When we assess children using dot cards. (Assessment 4. Number Arrangements) and ask, "What groups do you see", we can identify those children who do not yet see groups. Even after modeling what we mean by a group, children will respond with comments like "All of them." or "I see a line right there. Or they will say the total number of dots in the arrangement. This level of thinking is evident in other settings too. When children are making arrangements with six toothpicks, for example, the teacher might sit by the child and say "Oh, I see that your design has a 4 and a 2." When they don't yet see parts, children will respond, "No I don't have 4 and 2. I have 6. See, 1, 2, 3, 4, 5, 6." The children described above are not going to learn the parts of numbers until they can describe parts. They need something else that will help them pay attention to parts of numbers. They need a prerequisite.

The Assessing Math Concepts assessments determine the Instructional Levels for particular math concepts. If the child does not understand what is being asked, the Instructional Level is Needs Prerequisite (N). If they have an idea that is not fully developed, the Instructional Level is Needs Instruction (I). If they understand pretty well but they are not proficient yet, the Instructional Level is Needs Practice (P). If they understand the concept and are proficient, the Instructional Level is Ready to Apply (A).

We can describe the development of Critical Learning Phases using the same Instructional Levels. The children described above, who cannot yet see parts of numbers, would be at Instructional Level: Needs Pre-requisite (N). (In Range 1, the focus is on learning to recognize arrangements of numbers up to 5 which is a prerequisite concept. If children are going to be able to see, for example, a group of 4 as part of a larger number, they first need to be able to recognize a group of 4 by itself.)

Once children begin to see parts, they will be at the Needs Instruction (I) level. At this point, they are beginning to see some 2s and maybe sometimes a group of 3 in certain arrangements of numbers up to 6. (This is the focus for Range

2. They look for parts and, in the beginning, count all to find the total.)

The next Instructional Level is Needs Practice (P). At this point the children are getting better at seeing parts. They began seeing mostly 2s and 3s but over time they will also see groups of 4 and sometimes 5. And with practice, the groups become easier for them to identify. When the parts are obvious and the children no longer need to work hard to find them, they are at the Instructional Level: Ready to Apply (A).

2. Instructional Ranges are more specific than the Critical Learning Phases in that they identify the range of numbers to focus on.

When determining whether children understand and can apply any particular Critical Learning Phase, we cannot say a simple yes or no. It depends on the size of the numbers. Children may be "Ready to Apply" within one range of numbers yet "Need Instruction" or "Need Practice" when working with larger numbers. For example, if a child is developing proficiency learning the parts of numbers to 6, they are usually not yet able to easily find parts in larger arrangements. They need "Need a prerequisite" or "Need Instruction" with arrangements of 8 or more.

3. Each stage that children go through in learning math concepts requires a new level of insight.

When children are learning to add and subtract, there is a predictable path they will follow. Some move faster than others, but all must gain particular insights to move forward. If they are asked to work beyond their level of thinking, they will have to count or memorize "facts" they do not fully understand.

So far, we discussed the Critical Learning Phase:
"Recognizes and describes parts contained in larger numbers."

The next closely related Critical Learning Phase is:
"Describes parts of numbers, counts on to determine total."

Even though Counting On is often presented as a skill children can be taught, children will not actually use this strategy independently unless they understand what is happening with the numbers. Counting on becomes a useful strategy when children see (gain the insight) that when they already know a part, they don't need to start from one. Once they understand this, they will become more and more proficient with practice. They will move from counting all to sometimes counting on, to counting on easily when it makes sense to do so. The size of the numbers has an influence in this case too. The children may be able to count on from smaller numbers and not yet be able to for larger numbers.

The next related Critical Learning Phase is:
"Combines parts by using related combinations."

Counting on is an insight that is important to children learning to combine numbers. But it is important that they don't get stuck there. When they learned to count on by noticing for themselves that they didn't need to count from 1 every time they added, they will naturally move on to more sophisticated ways if they are looking for "easier" ways to add. So, a child might say, "I just knew that one. I didn't need to count or anything." So the focus in Range 3, is on helping children learn to combine parts of numbers to 7 without needing to count. Instead of counting, they will realize they "just know" for combinations like 3 + 3 and 4 + 1. They also learn they can use what they know to figure out what they don't know once they see the relationships.

When children are able to see relationships between combinations, they are able to learn several combinations at the same time. They will learn them because they see "It has to be that way." They notice that 3 + 4 = 7 so 7 - 4 leaves 3. 4 and 4 is 8 so 4 and 5 must be 9. I know that 3 and 3 is 6 so if I move 1 counter and make 4 and 2, I know it is still 6.

They will also be working with Numbers to 10 in Range 3 but will be combining these larger numbers by counting all or on.

4. Even though the Ranges identify the numbers most children will be focused on, the nature of the tasks allow children to work at their own level.

Within any classroom, the largest group of children will be working within the parameters of a particular range with some children working with concepts or numbers that come before and a few children working beyond the range. It is not necessary for teachers to separate children into groups for Number Talks because there will naturally be problems involving smaller and larger numbers. Suggestions for meeting the various needs are included in each chapter.

The following quote is from Number Talks in the Primary Classroom, p. 121

"Children in any classroom are at differing places as they move through the natural stages that occur on the way to developing proficiency. At this time, <u>most</u> children will be working at Range 3 and [while working at this range] will [eventually] know many of the combinations [to 7] without counting. If children are still thinking about numbers at the Range 2 level, they will [most often] count all to find out how many. Additionally, some children will be working beyond Range 3 and will either know the parts of numbers to 10 or easily use relationships to solve those they don't know."

5. The Instructional Ranges help ensure that teachers give children experiences and time they need to develop computational fluency.

We organized the mathematics into Instructional Ranges as a way of helping teachers focus on how and when computational fluency develops within particular ranges of numbers. Without this information, it is more likely that teachers will respond to the pressure to move on to larger and larger numbers before children can add and subtract without counting. What happens then is that too many children continue to count to solve problems for years to come with no chance to develop proficiency with the smaller but important foundational numbers. When teachers are aware of what children know (and don't know) within a range of numbers, they can base their instruction on what children can and cannot do and will give children the experiences that will help them move from counting to knowing without counting.

Once teachers know what to look for and which numbers are within the appropriate range, they will see a different level of engagement than they do when the numbers are too big to think about or too small to be interesting. There is an easy way to know when you have presented "just right" problems using "just right" numbers. All you need to do is look for the spontaneous smiles that reveal the sense of accomplishment children feel. ~ *Kathy*

If you're using Assessing Math Concepts and have a question regarding any of the nine assessments, we'd love to hear from you. Please email us your question to info@mathperspectives.com.

If you are using the paper Student Interview forms and would like to receive information on the Web-based version or professional development, please contact us at info@mathperspectives.com.

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