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Assessing & Developing Math Concepts

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Kathy Richardson is the author and developer of the Assessing Math Concepts (AMC) series of assessments and the Developing Number Concepts (DNC) series for Kindergarten through Second Grade Mathematics. Kathy, Program Director for Math Perspectives, is one of the most respected early childhood mathematics educators. Kathy answers questions from teachers across the country who are using AMC and DNC.

If you have questions for Kathy, please send them to Math Perspectives at info@mathperspectives.com.

ASSESSING MATH CONCEPTS: Two Digit Addition and Subtraction

Q Can you explain to my why touching the model automatically warrants "Needs a Prerequisite"? The question "how did you think about that" leads students (to use the model to explain how they thought about the problem. For example, if they broke up a number to make a ten, they might point and touch the model as they explain that. The final question then asks "Did the student point, touch or move the model to solve the problem?" The student may not have had to do this while solving, but may have used it to explain.

The "N" indicator suggests that the student is not ready for these ideas yet which in many cases is not true for this assessment. Shouldn't the N be for someone who is using it to count or breaking it up without regard to tens and ones or related combinations. This is not how the online tool assigns the N? - *Brookline, MA*

A The question, "Did the student point, touch or move the model to solve the problem?" does not refer to what the child did to explain how they solved the problem, but only if they needed to touch the model to find the answer. Teachers need to make sure they understand this or the student will end up with an N when they shouldn't. However, they can get an N even if they make tens and ones and understand how to add and subtract IF they need the model to act out the problem and find the answer.

Let me try to explain why touching or moving the model is so important that it warrants an N even if the student builds tens.

What we are trying to find out is whether the student is dependent on the model to get the answer. Are students able to think about making and breaking up tens without needing the model to act out the problems? Eventually, we want children to get to the place where they can work with symbolic problems in the same way they solved problems with the models. So, when they work with the models during instruction, the models need to be aids to thinking about what is happening when they add and subtract, not tools for getting answers or to demonstrate memorized procedures. The concrete models should help children develop more and more efficient strategies as they learn to take numbers apart and as they recognize particular relationships among the numbers. It is important when working with models during instruction that students think about the numbers and not go through the motions without paying attention to what is happening.

There are 4 stages of using models that children move through:

Four Stages of Using Models
Stage 1. The student needs to manipulate/touch the model.
Stage 2. The student can look at a model and mentally rearrange or move the model.
Stage 3. The student can refer to a partial model (just one number is represented with a model) and imagine the rest.
Stage 4. The student can think about what is happening when he looks at symbols. (And can use a model to show where the answer came from.)

Assessment 9 starts at Stage 3. Children are shown a partial model but are asked to think about what will happen when they add or subtract without touching the model. Some children are not yet able to work without the model and are at Stage 1. This means they need to work on understanding addition and subtraction until they reach the point they can work with a partial model.

However, if children are getting an N when it seems they shouldn't, we need to determine whether some children are able to solve the problem without the model, but like the security of using models and want to make sure they are right. If so, we might need to interrupt them if they begin to move the model and ask them to see if they can do it without the model. If they know the parts and understand what is going on they might glance at the model but not actually need to use it.

If they understand the underlying concept of adding by making tens and ones, but still need the model, the prerequisite indicated by the N in this case is that they need more experiences with the model where they focus on noticing what happens when they make and break up numbers to make tens. The next step would be to anticipate what is going to happen before they use the model to see if they visualized it correctly .

Some children may just need more time to be able to solve problems without a model. Perhaps others are using the models during instruction without thinking much about what they are doing - thus staying dependent on the model.

I hope this helps. Let me know if you have further questions and I will attempt to make my answers more clear.

~ Kathy

ASSESSING MATH CONCEPTS: Using AMC with 3rd and 4th Grade and Strategies

Q I am using the math assessments. You say they are for K-2 students. I have used them with 3rd and 4th grade students and feel that they show where the understanding breaks down. Am I wrong in using the assessments for that purpose? - *Aberdeen, SD*

A If you are getting information from the assessments that helps you identify the needs of your students, then that is a good use of the assessments. Many who teach 3rd and 4th grade find their students do not have a good understanding of place value even when working with tens and ones. It seems important to identify this so you can provide the students the experiences they need before they get farther and farther behind. Students often appear to understand in the earlier grade levels, but their lack of deep understanding shows up when they need to build on those understandings in order to move on.

If you have any other questions, let me know. *~ Kathy*

DEVELOPING MATH CONCEPTS: Learning Facts to 20

Q We are working on knowing math combinations. So many of our students can tell us strategies, but don't actually use those strategies when solving math problems. We are working with 2nd graders to know from memory addition/subtraction to 20. The strategies we focus on are making 10, doubles, doubles + or - 1, and breaking apart numbers. When we talk about strategies to use, they tell us how to use those I listed, but in their work they go back to fingers and counting on. We have used cubes, 2-sided counters, number lines, ten-frames, rekenreks and other manipulatives to help them see and understand strategies. We also use story problems. What can we do to help them actually use the strategies? - *Aberdeen, SD*

A When children learn to "do strategies" because the teacher teaches them, the strategies are often not useful to them. When children actually have to solve problems on their own, they tend to do whatever seems easiest to them at the time. For most kids, this will involve counting. I would recommend that we not teach strategies directly. The strategies that children use should come from them looking for and finding more and more efficient ways to solve problems as they learn more and more mathematics. The goal is not the strategy itself. The strategy is meant to help the children see the "easy" way to solve the problem.

The facts from 10 to 20 can be easy for children if they truly understand the structure of the numbers and the relationships inherent in those numbers. This means they need to think of these numbers as one ten and some more. They need to move from thinking about 17, for example, as 17 units to thinking of it as 1 ten and 7 ones. If they think of 17 as 1 ten and 7 ones, they can easily subtract the teen numbers 17, 16, 15, 14, 13, 12, 11 and 10 by taking the 1 ten away and then subtracting the ones. They can just as easily take away 7, 6, 5, 4, 3, 2 and 1 by leaving the ten and subtracting the ones. The only numbers that should cause them to stop and think are 8 and 9 because they have to consider how to take away more than the number of ones. If they think of 9 as 1 less than 10, then that becomes obvious to them too.

When adding, making a ten can be an easy way to combine numbers but only if they think of adding as making all the tens they can and seeing what is left over. So 8 plus 6 is easy if they think, " I need to get 2 out of the 6 to make a ten and then I will have 4 left. That is 1 ten and 4 ones... 14. BUT if they think of 17 as a pile of 17 ones or 14 as a pile of ones, they won't see the value of the ten and some more structure.

What children need to learn are the parts of numbers to 10 and then to think of numbers larger than 10 as 1 ten and some more. In addition to making and breaking up tens, children sometimes see the value of using near doubles to get an answer. They need to look at a problem like 7 + 6 and see the double that is "inside the numbers" that makes it easier to solve than counting on. Their focus should be on what they are doing with the numbers, not on what strategy they are using.

Children do not learn to think about numbers as tens and ones as quickly as adults would like them to. They need time building the numbers and making the tens. They also need time to break numbers apart and put them back together until they know the parts, not from memory but because that is the way that numbers work. For example, they need to think "I know 3 and 3 is 6 so 3 and 4 must be 7". Developing Number Concepts: Book 2 has a chapter that can give you some ideas on how to engage your students in learning about one ten and some more and using doubles.

What I would do if I saw my students automatically counting instead of looking for the easy way to find the answer, is to spend time giving them problems and then discussing what they did to find the answer. Number Talks are a particularly powerful setting for this to happen. I would use models to represent the numbers until students no longer need to refer to the model in order to think about the numbers. I would take whatever work they have been doing where you noticed they are not using strategies and have them see if they can find ways to get the answers that makes it easier to solve than counting. I would have an open discussion with them about what we are looking for. We want them to look at the numbers and see that they don't need to count because it is easier to

make a ten or break a number into one ten and some more. The key here is that children are doing the thinking. This is different than trying to remember what the teacher wants them to do. They need to think about the numbers and not the strategies. They need to know they are trying to find ways that make it easy to find the answers... not trying to apply the strategy they think the teacher wants them to use.

I hope this response gives you something to think about and try with your students. Let me know if you have any more questions and what happens when you ask kids to find easy ways to solve problems besides counting. I am very interested in what you discover about your students as they share their thinking with you.

Best, *Kathy*

DEVELOPING MATH CONCEPTS: Place Value

Q What is the PURPOSE of unitizing numbers such as 4,5,6 in the zib (PLUS ONE MINUS ONE) game in Book 3 of Developing Number Concepts series (Place Value, Multiplication and Division)? How does it help children with understanding place value or with the process of regrouping? - *Hillside, Illinois*

A Thank you for your question. Place value is a really important concept for children to understand. They often move through school not really understanding what is going on - only knowing how to name the place value of the digits. But what they need to understand is that numbers are composed of tens and ones, and hundreds, tens and ones and so on. In order to understand the structure of numbers as tens and ones, they need to understand that we count groups as though they were single objects. This is not obvious to young children who are just moving away from thinking of numbers like 7 as a collection of single units. Even children who have worked for a long time with place value presented in the traditional way, still have trouble unitizing. They continue to think of 6 tens as made up of 60 units. I found an interesting quote from researchers in Australia that confirms what I have seen in children.

"Many students who are able to identify place-value parts (eg, they can say that there are 4 hundreds 6 tens and 8 ones in 468) and count orally to 100 and beyond, still think about or imagine 2 and/or 3 digit collections additively in terms of ones (ie, 468 is actually understood as the sum of 400 ones, 60 ones and 8 ones)." - *Department of Education and Early Childhood Development: State of Victoria, Australia*

The Grouping Games in Book 3 have the children forming and counting groups to help them develop an understanding that a numbers describe groups and these groups can be counted as one entity (unitized) . Instead of saying " the 3s" or the 4s, the class makes up a name to call each group– something like "wob" or zork" We use smaller numbers because children can make the groups quickly and thus more easily focus on the process of making and counting groups. We do it with several numbers so they can generalize these ideas and not just think of it as a "game." While they are making the groups, they record as they go. This helps them see the patterns and also helps them understand that a 17 is not a collection of 17 ones, but is 1 group of 10 and 7 leftover ones. When they are recording the patterns of 4, for example, they would start by writing 0 "wobs" and 0. As they add one at a time, they will record 0 "wobs" and 1, 0 "wobs" and 2 all the way to 1 "wob" and 0 which would look like 10, then 1 "wob" and 1 which looks like 11 and so on. Teachers often get worried about this as they think it will confuse children. But instead, it really helps make the point that the first number describes the number of groups and the second number describes the number of ones. All you need to do is remind them that when they are working with "wobs" the first number tells you how many "wobs" you have- not how many tens. After a period of time working with forming groups of 3s, 4s and 5s, children understand what the numbers stand for. Then when you go on to tens, they really get it. They know the first number tells how many tens! Doing the grouping games with my second graders made all the difference in the world. It may look confusing but it is really a powerful way to make tens and ones meaningful. When children understand the process of making tens and think of the numbers as tens and ones, then adding and subtracting 2 -digit numbers makes sense to them. They still need lots of time working with tens and ones and adding and subtracting as described in Book 3 to develop proficiency, but the Grouping Games really help them understand what is happening with the quantities they are working with and what the digits represent.

Hope this is helpful. ~ *Kathy*

If you're using Assessing Math Concepts and have a question regarding any of the nine assessments, we'd love to hear from you. Please email us your question to info@mathperspectives.com.

If you are using the paper Student Interview forms and would like to receive information on the Web-based version or professional development, please contact us at info@mathperspectives.com.