



## *Stay Connected!*

Kathy Richardson is the author and developer of the Assessing Math Concepts (AMC) series of assessments, the Developing Number Concepts (DNC) series for Kindergarten through Second Grade Mathematics and Number Talks for the Pre-K and Primary Classroom. Kathy, Program Director for Math Perspectives, is one of the most respected early childhood mathematics educators.

Kathy answers questions from teachers across the country who are using AMC, DNC and Number Talks. If you have questions for Kathy, please send them to Math Perspectives at [info@mathperspectives.com](mailto:info@mathperspectives.com).

### QUESTION

Hi Kathy! A few things have come up recently in analyzing assessments and I figured I would reach out and ask the expert herself! No rush on these; thank you in advance for your thoughts.

1. When assessing using More/Less Trains, sometimes students can answer with the right number but use the wrong language. For example, children will respond to “how many less” by saying “3 more” or “3 more less.” Would you consider this “ready to apply”?
2. When assessing ten frame addition and subtraction – I’ve had a few children (who knew parts of numbers to 10 with fluency) quickly answer the questions incorrectly, for example  $8 + 6$  they would say 2 to make a ten and 3 leftovers, 13. When it’s this quick and confident I’ve asked the child what number they think I added and in this example the child said 5. When I corrected them and told them I wanted them to add 6, they were able to immediately correct their answer to 4 leftovers, 14. Does this still count as “ready to apply”, or would you think that a child needs to be able to retain the number added or realize on their own they don’t know it and ask again?  
Math Tutor, NC

### ANSWER

I love hearing from you. I hope you will continue to ask questions or share stories about the kids.

**Q. 1:** The More/Less Trains assessment is intended to find out if the children can see the differences between quantities and figure out how many more or less. I accept imprecise language like there is “more less” because I am interested in finding out if they are able to notice the relationships at that level. When they can see and describe the differences, I know they can benefit from working with comparing numbers, with the end goal being knowing the differences between numbers without counting.

How children use the language is not part of the assessment but still important as they work to compare numbers. Keep modeling the correct language when opportunities arise. Make note of which children use imprecise language and check in to see if they begin to say “less” or “fewer”. They need to learn the correct language before you can leave that concept totally behind.

**Q. 2:** About the ten frames. I am curious about the children you didn’t describe who give fast answers that are wrong. There is an underlying concept that we need to be aware of. The key here (besides being able to break numbers apart) is whether or not they are thinking of ten as a unit. Some children will tell you how many to make a ten and how many left but think of the problem as  $8 + 2 + 3$ . If that is the case, they may be adding all 3 numbers or counting on from 8. When adding 3 numbers it is easy to make a mistake. If instead, they are thinking, “I made a ten.” The 2 is now part of the ten and handled, so to speak. All they have to do is add on the 3. It becomes almost obvious at that point. That seems to be a

different issue than forgetting the number they were asked to add. In the case of the child you described, it seems they did just forget the number. In that case, I would say it was correct.

## QUESTION

Good evening, My child goes to a school that uses Investigations Math and the Kathy Richardson Assessing Math Concepts. My child was a remote learner all year, so the assessment was done via iPad. It was over the course of two days and I could hear the assessment happening in the other room. I am also a teacher myself, certified in elementary grades 1-6 and mathematics grades 1-6.

I was pretty baffled that the first part of an end of year second grade assessment had them playing "what's hiding" for facts up to 10...as it is a first grade concept and standard in the state of MA. Actual direct/explicit instruction was very limited this year...my child had to figure out addition and subtraction on their own as they were never taught strategies to use to solve 2 digit math problems. My child actually started using an ineffective strategy I had to correct as it yielded incorrect answers every time, to which the school never corrected the ineffective strategy. Apparently they assume children learn all math concepts through osmosis...

My concern and need for clarification arises when during the second day of the assessment my child had a mini whiteboard and marker out to keep their thoughts organized...my child by no means uses the standard algorithm to solve anything but likes to write down their thinking as they are a tactile/kinesthetic learner...solving methods for solving are like the examples below:  $26+25 \rightarrow 26 + 20 = 46, 46+ 5 = 51$  and for subtraction  $87- 29 \rightarrow 87-20=67, 67-9=58$ .

The problems presented to my child were two 2-digit numbers that included regrouping and no regrouping for both addition and subtraction problems. My child was told they were not allowed to write anything down but had to do all their thinking mentally in their head by the math coach/specialist teacher. Is this what was supposed to happen? All year they have been requiring students to show their thinking by writing it down and explaining how they got their answers (it is good math practice as it is something I do in my own classroom) but during a major assessment they are no longer allowed to write down their ideas to keep their thinking straight? How is that valid?

Of the assessment results I was informed with the following (and not what it actually means thus my reaching out to you and having to do my own research)... "...and 'P-Needs Practice' with solving problems without a model and solving symbolic problems." What does this mean? Does this mean the ability of solving addition/subtraction problems in your head? Or struggles to solve uses methods like what my child does (see examples above)? My child does not struggle to solve addition/subtraction problems (with and without regrouping) when they are allowed to write down their thinking as shown above, unfortunately my child was not allowed to do that during the assessment, as I have mentioned.

Would you be able to clarify all of this for me, please? I would greatly appreciate it.  
Thank you. *Parent of Second Grade Student, MA*

## ANSWER

Hello, Thank you for your interest in learning more about the assessments and seeking to understand what teachers were trying to find out when using them as they did.

I will do my best to describe the essence of these assessments and explain why certain decisions were made in determining how the assessments should be presented to children. I have written longer explanations in the assessment books and in my book about How Children Learn Number Concepts, a Guide to the Critical Learning Phases. But I think I can share enough of my thinking for you to get some understanding of what the assessments do. And of course, if you are interested, I am happy to answer any questions that come up for you after reading what I have to say.

There are foundational understandings about numbers and how they work that children need to know in order to build on these concepts as they move on in school. Children develop an understanding of number concepts over time, going through certain levels of thinking as they go. For example, at one stage of thinking, young children see each number as separate from other numbers and can't see the smaller numbers within the larger numbers. At another time, children will not be able to see that organizing quantities into tens and leftover ones tells you how many without needing to count them. The level of thinking a child has reached influences what they are able to learn at a particular time. I call these levels of thinking or insights Critical Learning Phases.

Most assessments are directed at finding out whether children know how to "do the math" and get right answers. The problem is that children's ability to get answers often hides the level of understanding they have reached and leads to the eternal problem of too many children not understanding math when they get to middle or high school.

The Assessing Math Concepts assessments are focused on (1) the thinking behind the answers and (2) the level of proficiency they have reached. Instead of grading the assessment in a traditional way, we determine the instructional level.

N Needs Prerequisite

The child does not understand the concept and needs some different experiences.

I Needs Instruction

The child has a very beginning sense of the concept. I think of it as "having an inkling."

P Needs Practice

The child understands what is going on but working with the math is not easy yet so he would benefit from more practice.

A Ready to Apply

The child understands what they are doing and the math is easy for him so he doesn't need to work on these types of problems anymore.

The Two-digit Addition and Subtraction assessment is intended to see if the student is able to use what they know about tens and ones to add and subtract. We want to find out how meaningful the numbers are to them, so the questions refer to the models rather than just the numbers. The first problem is  $28 + 16$ . The child is shown 2 Unifix Trains of ten and 8 ones and tells how many he sees. The next question is "If you were to add 16 to the 28, how many tens and how many ones would there be?" The question is not "What is the answer?" but rather is focused on how the child determines the number of tens.

The child can look at the model of 28 but has to imagine the 16. Some typical responses would be: "I can add the 10 from the 16 and that would be 3 tens. Then I have to add 8 and 6. I can take 2 from the 6 and that makes another ten so now I have 4 tens and 4 left over." Another response might be: "I have 2 tens and 8 and I add another ten, so I have 3 tens and 8. I still need to add 6 more. I can take 2 from the 6 to make another ten and I have 4 left over. So, I have 4 tens and 4 ones."

Children who are thinking of the model and adding a 10 and a 6 do not typically need to write down the steps because they are "gathering tens" as they go. I can see that if your son was used to writing down the steps, he may have been taken aback when he was told he couldn't write and then had trouble focusing on what the question was. OR he could be used to combining symbols and isn't thinking in terms of groups of ten. One way to find out is to have him show you his steps by moving the model of the tens and ones to show what he is doing. That will give you some insight into the way he is thinking about the numbers once he understands the question he is being asked. If it seems like it would be helpful, you may have him use models for both numbers at first and then move to imagining the second number.

The assessment has 2 opportunities to work with a visible model. The third question still refers to the model but the child is asked to try to solve the problem without looking at the model. First, they

are shown the model and tell how many and then it is covered. The same question is asked. There is a point when some children need to look at the model in order to do the problem so they can lift the paper if they need to.

The last problem is a written problem but the question is still asked in terms of what is represented by the numbers.

28 + 24 "If you add 24 cubes to the 28, how many tens and ones will there be" .. How much is that altogether?

There are 3 levels identified in the assessment

1. Solving problems with the model
2. Solving problems without the model
3. Solving symbolic problems.

When children learn to solve problems without thinking about the numbers and what they represent, we find they cannot build on what they know. They often have misconceptions about what is going on with the numbers. Some of them will build 28 using 2 cubes for the 2 tens and 8 cubes for the 8 ones. They will move 1 cube to carry the ten. Some will call it a one and some will call it a ten but still represent it with 1 cube.

Sometimes children think of paper and pencil procedures rather than thinking about the meaning. Recently a fourth grader added 10 to a number and explained it this way: "8 plus 0 is 8. And 2 + 1 is 3 so the answer is 38. Rather than thinking about adding 1 more ten, they had to add it like any other math problem they have solved before.

Children who have had lots of experiences and think of numbers as groups of tens and leftover ones, see the relationships and solve problems using those relationships. For example, I showed a 2nd grader 2 tens and 6 ones and asked him how many there would be if he added 7. He mentally took 4 from the 7 to complete the ten and knew he had 3 left so saw there were 33 very quickly. I then asked, "What would it be if you added 17". He instantly said "43". I asked him how he knew that so quickly He said, "Because you just add another ten." He is thinking of numbers as tens and ones so it was obvious to him that the answer had to be ten more.

Children know so much more than how to get answers when they understand the numbers and how they are structured. They can apply what they know to larger numbers rather than having to learn what to do to get the answer.

Knowing the parts of numbers and seeing how they are related to other parts will help children deal with larger and larger numbers with ease. We want more than memorizing the parts. We want them to use what they know to figure out what they don't know. For example, if a child not only knows 4 and 4 is 8 but also sees that moving 1 of the cubes to show 5 + 3 will also be 8 makes knowing parts so much more useful than if they just memorized them. They will be able to use this to see that 15 + 3 must be 18, that 1541 + 319 must be 1800 plus some more and so on.

We have found that most children are still working to learn the parts of numbers within 10 through 2nd grade. Most can easily figure out the answer, but it will serve them well to be so familiar with the parts that they can add and subtract multi-digit numbers without having to stop and figure anything out. It is not unusual for 2nd grade teachers to give the Hiding Assessment to any students who are still working to gain proficiency. I am glad to see these teachers moved on to Place Value concepts whether the students knew all the parts to 10 or not.

I don't know if I have completely answered your questions or not. Please feel free to continue the conversation if you think it will be helpful to you.

*Best, Kathy Richardson*

## QUESTION

Hi Kathy, I'm a math specialist and I have 2 questions for you. My PLC partner and I are working with some students (1st-3rd) on making a pile. We see them for 15 min 4 days a week. Students did not have additional math services last year b/c of the pandemic. We have been working on these skills since mid-October.

We are noticing some kids are having trouble holding the number in their head (usually above 8). They will count past it. As we read your book *How Children Learn Number Concepts*, a few quotes stood out to us...

"A number must have enough meaning to children so that they can remember the number while they are counting."

So we are wondering, how do we give meaning to numbers? Is it putting it into context? Is it more practice? Is it having them say the number a few times before they start to make the pile?

If they can't do, "the next stage is children hearing themselves say the number asked for and remember to stop at the particular number," how should we respond? Do we help students self correct? Do we make them aware of their error and give them feedback? Do we have them start over and try again? We are trying to work carefully within the numbers they need to work on.

Thank you for your help and expertise! I'm so thankful for you. I hope you have a great week!  
*Sincerely, Math Specialist, Minnesota*

## ANSWER

Hi there, Thank you for your caring and thoughtful questions. Your questions and those from other teachers help me realize I need to explain a different way of thinking about how to help children who are having difficulty with a particular idea. Basically, I want us to think, "Why can't they do it?" rather than "How can I get them to do it?" We are so used to thinking that we need to give kids more practice if they are having trouble. Or we need to change the question or intervene in some way so they can get it "right." But I think we need to give them the experiences that will lead them to understanding what you are asking them to do. It means we work on a prerequisite concept rather than working on what they are having trouble with.

It is helpful to recognize the difference between "insight" and "proficiency." When children do not have the necessary insight, practice does not help them. After they understand, practice will help them get better at it. So in this case, if the child cannot keep a number in his head because he lacks meaning for the number, then we need to help the numbers become meaningful. That means working with the kinds of activities found in *Developing Number Concepts, Book 1*. It is the variety of experiences where they work with the numbers in different ways that will develop meaning. So if a child was working with Shape Puzzles, for example, they would first say how many cubes they think will fit in the shape. Then they will actually find out.

You might start with Numbers to 10 and move down to Numbers to 6 or on to Numbers to 20. Whatever you decide, you need to watch what they do. If you ask, "How many do you think will fit?", do they shrug to say they have no idea? Do they make a wild guess and just say any number that comes to mind or do they look carefully at the shape and make a reasonable guess? If they have no idea or make guesses, work with smaller numbers. If they have some idea, watch them doing a variety of tasks to confirm and then move to larger numbers. They should experience many different arrangements and situations using activities like Line Puzzles, How Long Is it? And so on.

You can also check to see what size number they can keep in mind. The assessment is a check that will give you an idea of what range of numbers you should work on but during instruction time, you can try lots of different numbers. Activities that ask them to count out a particular number can help

them think about what they are counting. You can work with Grow and Shrink, for example, and make the numbers as small or as large as needed. Roll A Tower also asks children to keep a number in mind.

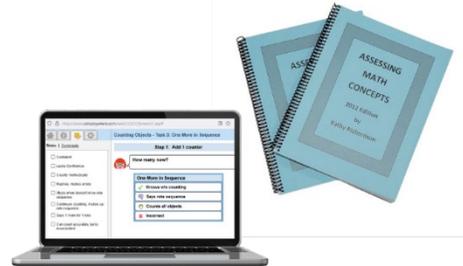
So I recommend taking the time to make numbers meaningful and when they can, the children will make a pile without any extra help from you. I know time pressures make you feel you don't have time to "go back" but the right experiences will help them make a leap and do more in the long run.

Let me know if you have any further questions about what I wrote or about what happens when you work with your children. I would love to know more about their progress as well as your frustrations and successes as you work to provide what they need.

*Kathy*



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If you're using Assessing Math Concepts and have a question regarding any of the nine assessments, we'd love to hear from you.

If you are using the paper Student Interview forms and would like to receive information on the Web-based version or professional development, please contact us by emailing [info@mathperspectives.com](mailto:info@mathperspectives.com) or call **360-715-2782**