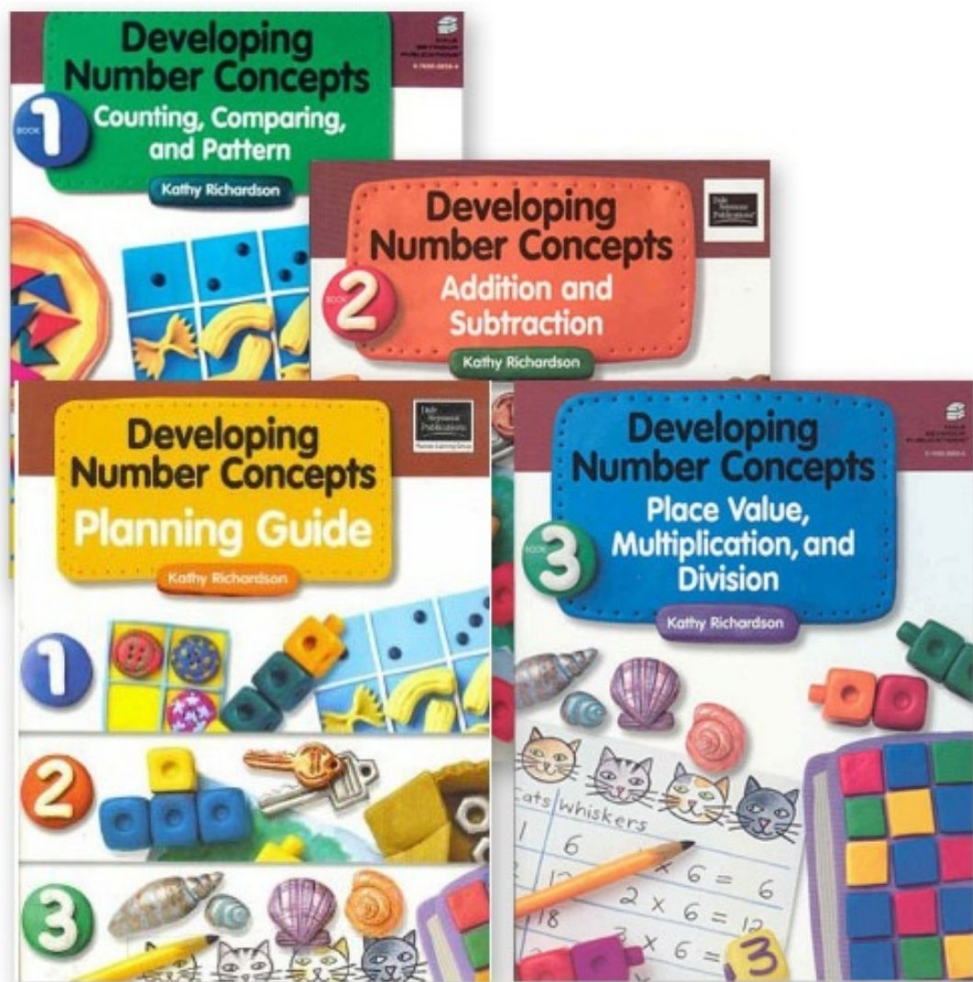


# *The Research Basis for the* **Developing** **Number Concepts** *Series*





# **The Research Basis For the Developing Number Concepts Series by Kathy Richardson**

## **Research Findings**

The research document by Tracy Hall, Ph.D., Senior Research Scientist for the National Center on Assessing General Curriculum states that not all students are alike. Based on this knowledge, differentiated instruction applies an approach to teaching and learning so that students have multiple options for taking in information and making sense of ideas. The model of differentiated instruction requires teachers to be flexible in their approach to teaching and adjusting the curriculum and presentation of information to learners rather than expecting students to modify themselves for the curriculum. Classroom teaching is a blend of whole-class, group and individual instruction. Differentiated Instruction is a teaching theory based on the premise that instructional approaches should vary and be adapted in relation to individual and diverse students in classrooms.

## **Definition**

To differentiate instruction is to recognize students varying background knowledge, readiness, language, preferences in learning, interests, and to react responsively. Differentiated instruction is a process to approach teaching and learning for students of differing abilities in the same class. The intent of differentiating instruction is to maximize each student's growth and individual success by meeting each student where he or she is, and assisting in the learning process.

## **Identifying Components/Features**

According to the authors, several key elements guide differentiation in the education environment. Tomlinson (2001) identifies three elements of the curriculum that can be differentiated: Content, Process, and Products. Additionally, several guidelines are noted to help educators form an understanding and develop ideas around differentiating instruction.

## **Content**

- *Several elements and materials are used to support instructional content.* These include acts, concepts, generalizations or principles, attitudes, and skills. The variation seen in a differentiated classroom is most frequently the manner in which students gain access to important learning. Access to the content is seen as key.
- *Align tasks and objectives to learning goals.* Designers of differentiated instruction determine as essential the alignment of tasks with instructional goals and objectives. Goals are most frequently assessed by many high-stakes tests at the state level and frequently administered standardized measures. Objectives are frequently written in incremental steps resulting in a continuum of skills-building tasks. An objectives-driven menu makes it easier to find the next instructional step for learners entering at varying levels.
- *Instruction is concept-focused and principle-driven.* The instructional concepts should be broad based and not focused on minute details or unlimited facts. Teachers must focus on the concepts, principles and skills that students should learn. The content of instruction should address the same concepts with all students but be adjusted by degree of complexity for the diversity of learners in the classroom.

## **Process**

- *Flexible grouping is consistently used.* Strategies for flexible grouping are essential. Learners are expected to interact and work together as they develop knowledge of new content. Teachers may conduct whole-class introductory discussions of content big ideas followed by small group or pair work. Student groups may be coached from within or by the teacher to complete assigned tasks. Grouping of students is not fixed. Based on the content, project, and on-going evaluations, grouping and regrouping must be a dynamic process as one of the foundations of differentiated instruction.
- *Classroom management benefits students and teachers.* Teachers must consider organization and instructional delivery strategies to effectively operate a classroom using differentiated instruction. Carol Tomlinson (2001) identifies 17 key strategies for teachers to successfully meet the challenge of designing and managing differentiated instruction in her text How to Differentiate Instruction in Mixed-Ability Classrooms, Chapter 7.

## Evidence of Effectiveness

Differentiation is recognized to be a compilation of many theories and practices. Based on this review of the literature of differentiated instruction, the "package" itself is lacking empirical validation. There is an acknowledged and decided gap in the literature in this area and future research is warranted.

According to the proponents of differentiation, the principles and guidelines are rooted in years of educational theory and research. For example, differentiated instruction adopts the concept of "readiness". That is the difficulty of skills taught should be slightly in advance of the child's current level of mastery. This is grounded in the work of Lev Vygotsky (1978), and the zone of proximal development (ZPD), the range at which learning takes place. The classroom research by Fisher et al. (1980), strongly supports the ZPD concept. The researchers found that in classrooms where individuals were performing at a level of about 80% accuracy, students learned more and felt better about themselves and the subject area under study (Fisher, 1980 in Tomlinson, 2000).

Other practices noted as central to differentiation have been validated in the effective teaching research conducted from the mid 1980's to the present. These practices include effective management procedures, grouping students for instruction, and engaging learners (Ellis and Worthington, 1994).

While no empirical validation of differentiated instruction as a package was found for this review, there are a generous number of testimonials and classroom examples authors of several publications and Web sites provide while describing differentiated instruction. Tomlinson reports individual cases of settings in which the full model of differentiation was very promising. Teachers using differentiation have written about improvements in their classrooms. (See the links to learn more about differentiated instruction).

Research shows that a critical factor correlated with positive educational outcomes is providing differentiated instruction matched to the level of skill development of the learner (Allinder 2000; Stecker 2000; Ysseldyke, 2003).

The National Council of Teachers of Mathematics (NCTM) identified five key content standards and five process standards as important learning phases that must be in place in order for children to be successful in math. The other four content areas, as well as the five process standards are based in the core area of number (NCTM, 2000).

In these Standards, understanding number and operations, developing number sense, and gaining fluency in arithmetic computation form the core of mathematics education for the elementary grades.

Counting is a foundation for students' early work with number p. 79 NCTM standards

Knowing basic number combinations—the single-digit addition and multiplication pairs and their counterparts for subtraction

and division—is essential. Equally essential is computational fluency—having and using efficient and accurate methods for computing. p. 32

The Number and Operations Standard describes deep and fundamental understanding of, and proficiency with, counting, numbers, and arithmetic, as well as an understanding of number systems and their structures... Central to this Standard is the development of number sense—the ability to decompose numbers naturally, ...use the relationships among arithmetic operations to solve problems, understand the base-ten number system, estimate, make sense of numbers, and recognize the relative and absolute magnitude of numbers (Sowder 1992). In NCTM Standards p. 32

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

Understanding of number develops in pre-kindergarten through grade 2 as children count and learn to recognize "how many" in sets of objects. A key idea is that a number can be decomposed and thought about in many ways. For instance, 24 is 2 tens and 4 ones and also 2 sets of twelve. Making a transition from viewing "ten" as simply the accumulation of 10 ones to seeing it both as 10 ones *and* as 1 ten is an important first step for students toward understanding the structure of the base-ten number system (Cobb and Wheatley 1988). Throughout the elementary grades, students can learn about classes of numbers and their characteristics, such as which numbers are odd, even, prime, composite, or square.

Representing numbers with various physical materials should be a major part of mathematics instruction in the elementary school grades. p. 33

As students gain understanding of numbers and how to represent them, they have a foundation for understanding relationships among numbers. p. 34

### **Compute fluently and make reasonable estimates**

Developing fluency requires a balance and connection between conceptual understanding and computational proficiency. On the one hand, computational methods that are over-practiced without understanding are often forgotten or remembered incorrectly (Hiebert 1999; Kamii, Lewis, and Livingston 1993; Hiebert and Lindquist 1990). On the other hand, understanding without fluency can inhibit the problem-solving process (Thornton 1990). As children in prekindergarten through grade 2 develop an understanding of whole numbers and the operations of addition and subtraction, instructional attention should focus on strategies for computing with whole numbers so that students develop flexibility and computational fluency. Students will generate a range of interesting and useful strategies for solving computational problems, which should be shared and discussed. At the grades 3–5 level, as students develop the basic number combinations for multiplication and division, they should also develop reliable algorithms to solve arithmetic problems efficiently and accurately. These methods should be applied to larger numbers and practiced for fluency. p. 35

**Instructional programs from prekindergarten through grade 12 should enable all students to—**

### **Developing Number Concepts Series**

**The concepts and skills related to number and operations are a major emphasis of mathematics instruction in pre-kindergarten through grade 2 p80**

**In prekindergarten through grade 2 all students should—**

- **count with understanding and recognize "how many" in sets of objects;**
- **use multiple models to develop initial understandings of place value and the base-ten number system;**

- **develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections;**
- **develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers;**
- **connect number words and numerals to the quantities they represent, using various physical models and representations;**
- **understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations;**
- **understand the effects of adding and subtracting whole numbers;**
- **develop and use strategies for whole-number computations, with a focus on addition and subtraction;**
- **develop fluency with basic number combinations for addition and subtraction;**
- **use computation, estimation, paper and pencil, and calculators. a variety of methods and tools to compute, including objects, mental**

Researchers and experienced teachers alike have found that when children in the elementary grades are encouraged to develop, record, explain, and critique one another's strategies for solving computational problems, a number of important kinds of learning can occur (see, e.g., Hiebert [1999]; Kamii, Lewis, and Livingston [1993]; Hiebert et al. [1997]). P.35 in NCTM Standards

Algebra Standards

Understand Patterns, Relations, and Functions

Use mathematical models to represent and understand quantitative relationships. P. 37 NCTM stand

Acting out story problems with cubes

### **Description of Developing Number Concepts Series**

The *Developing Number Concepts* series for Pre-K to 2<sup>nd</sup> grade mathematics includes the following three books and Planning Guide:

Book 1: *Counting, Comparing and Pattern*

Book 2: *Addition and Subtraction*

Books 3: *Place Value, Multiplication and Division*

*Planning Guide*: includes comprehensive year-long teaching plans along with classroom management ideas.

The series provides everything teachers require to meet the range of needs in their classrooms and ensure success for all students. The focus is on understanding and competence for all learners. Each chapter of each book includes Chapter Overview, Goals, Analyzing and Assessing Needs, Classroom Scenes, About the Activities, Teacher Directed and Independent Activities.

### **Application of Research**

**The Developing Number Concepts incorporates all of the ideas of the above research and ensures that differentiation instruction is possible for teachers to attain. The series:**

- *Clarifies key concepts* to ensure that all learners gain powerful understandings that serve as the foundation for future learning. Teachers are encouraged to identify essential concepts and instructional foci to ensure all learners comprehend.
- *Uses assessment as a teaching tool to extend versus merely measure instruction.* Assessment should occur before, during, and following the instructional episode, and help to pose questions regarding student needs and optimal learning.
- *Emphasizes critical and creative thinking* as a goal in lesson design. The tasks, activities, and procedures for students require that students understand and apply meaning. Instruction requires supports, additional motivation, varied tasks, and materials for different students in the classroom.
- *Engages all learners.* Includes activities that are engaging and motivating for a diverse class of students. Tasks vary within instruction as well as across students.
- *Initial and on-going assessment of student readiness and growth are essential.* Meaningful pre-assessment leads to functional and successful differentiation. Incorporates pre and on-going assessment that informs teachers to direct their instruction.
- *Students are active and responsible explorers.* Teacher's respect that each task put before the learner will be interesting, engaging, and accessible to essential understanding and skills. Each child feels challenged.
- *Vary expectations and requirements for student responses.* Students have varied means of expression, and are provide with varying degrees of difficulty.

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